Arrows

$\xrightarrow{0}$	$:=$	$\{(x,x) \mid x \in A\}$	**identity**
$\xrightarrow{i+1}$	$:=$	$\xrightarrow{i} \circ \rightarrow$	$(i+1)$-**fold composition**$, i \geq 0$
$\xrightarrow{+}$	$:=$	$\bigcup_{i>0} \xrightarrow{i}$	**transitive closure**
$\xrightarrow{*}$	$:=$	$\xrightarrow{+} \cup \xrightarrow{0}$	**reflexive transitive closure**
$\xrightarrow{=}$	$:=$	$\rightarrow \cup \xrightarrow{0}$	**reflexive closure**
$\xrightarrow{-1}$	$:=$	$\{(y,x) \mid x \rightarrow y\}$	**inverse**
$\leftarrow$	$:=$	$\xrightarrow{-1}$	**inverse**
$\leftrightarrow$	$:=$	$\rightarrow \cup \leftarrow$	**symmetric closure**
$\xleftrightarrow{+}$	$:=$	$(\leftrightarrow)^{+}$	**transitive symmetric closure**
$\xleftrightarrow{*}$	$:=$	$(\leftrightarrow)^{*}$	**reflexive transitive symmetric closure**

Terminology

- x is **reducible** iff there is a y such that $x \to y$.
- x is **in normal form** (**irreducible**) iff it is not reducible.
- y is **a normal form of** x iff $x \xrightarrow{*} y$ and y is in normal form. If x has a uniquely determined normal form, the latter is denoted by $x{\downarrow}$.
- y is a **direct successor** of x iff $x \to y$.
- y is a **successor** of x iff $x \xrightarrow{+} y$
- x and y are **joinable** (**have a common reduct**) iff there is a z such that $x \xrightarrow{*} z \xleftarrow{*} y$, in which case we write $x \downarrow y$.

More terminology

A reduction $\rightarrow$ is called

Church-Rosser	iff	$x \overset{*}{\leftrightarrow} y \;\Rightarrow\; x \downarrow y$
confluent	iff	$y_1 \overset{*}{\leftarrow} x \overset{*}{\rightarrow} y_2 \;\Rightarrow\; y_1 \downarrow y_2$
terminating	iff	there is no infinite descending chain $a_0 \rightarrow a_1 \rightarrow \cdots$
normalizing	iff	every element has a normal form.
convergent	iff	it is both confluent and terminating.