Asynchronous processing of proof documents – rethinking interactive theorem proving

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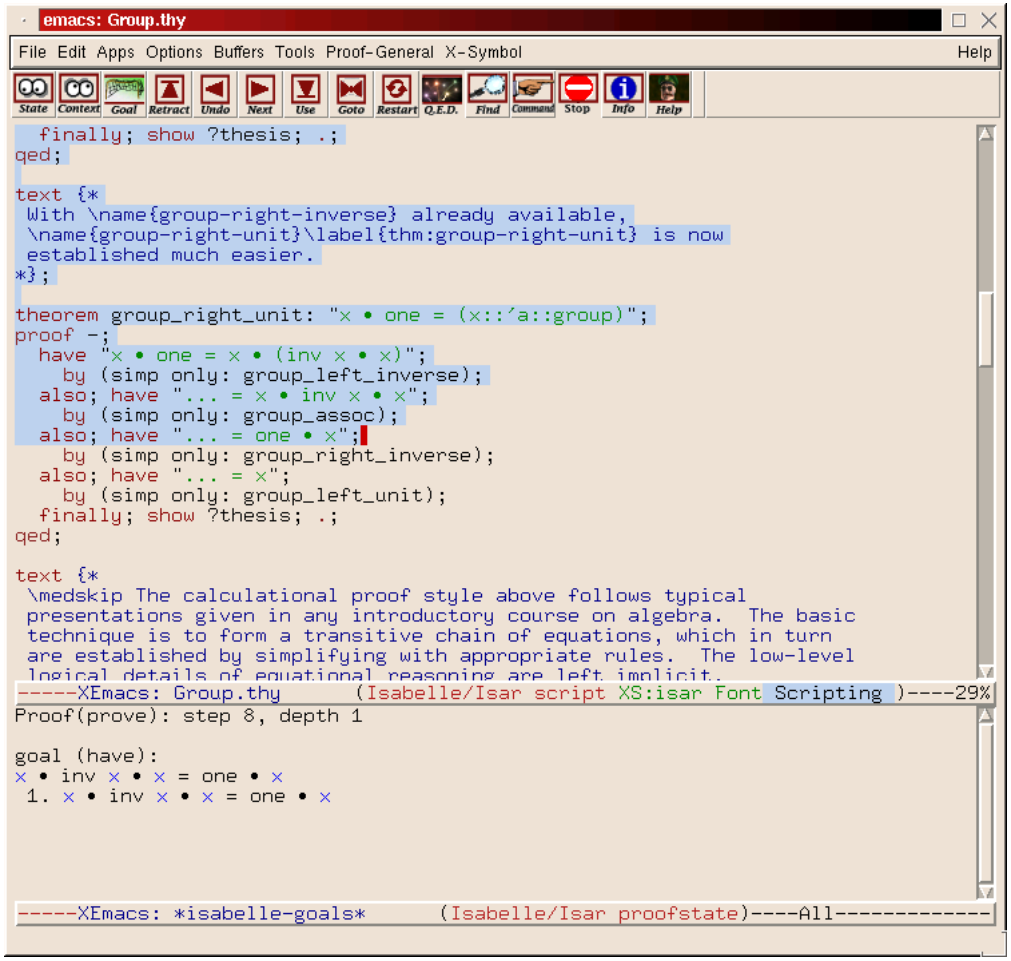
Motivation

General aims

- Support interactive development of larger formal theories
- Reduce requirements on front-end (editors, web clients etc.)
- Exploit parallel proof checking
  (multiprocessing is the elephant in the room)
- Exploit inherent structure of documents
  (implicit: proof irrelevance, explicit: Isar)

$\longrightarrow$ Towards the next generation of interactive proof checking

Example: Proof General



Main characteristics:

- sequential checking of *proof scripts*
- one frontier between checked/unchecked
- one proof state
- one response
- mostly synchronous (interface may block)

Example: Mizar

```
for A,B,C being set holds
  A c= B implies A /\ C c= B /\ C
proof
  let A, B, C be set;
  assume subset: A c= A;
::>    *52
  thus A /\ C c= B /\ C
  proof
    let x be set;
    assume a1: x in A /\ C;
    then x in A;
::>           *4
  end;
::> *70
end;

::> 4: This inference is not accepted
::> 52: Invalid assumption
::> 70: Something remains to be proved
```

Main characteristics:

- simultaneous checking of *proof text*
- always fully checked, potential omissions
- no proof state
- inline response
- mostly "batchmode"

Proof document processing

Isabelle language layers

Primitive layer: logic implementation

- Isabelle/Pure logical framework (as LCF-style kernel)
- Isabelle/ZF, HOL, HOLCF, . . . object-logics

Primary layer: Isabelle/Isar theory and proof language

Presentation layer: LaTeX generated from formal theory sources

Example presentations:

- → This ← (slides)
- http://isabelle.in.tum.de/dist/library/HOL/Unix/document.pdf (proof document)

The primary "document" model

Fundamental entities:

- *printed document*: result of processing a *session*
- *session*: graph of *theory* nodes
- *theory*: sequence of *commands* (transactions)
- *command*: theory specification element (definition, statement), or proof element etc.

Existing technology:

- sequential processing of command transactions
- synchronous reporting of success / error
- single $undo$ / $redo$
  Note 1: Relative state addressing, expressed as unary offsets!
  Note 2: Proof General only uses $undo$.

Example

datatype $foo = Foo \mid Bar\ foo$

lemma
  fixes $x :: foo$
  shows $P\ x$
proof $(induct\ x)$
  case Foo
  then show $P\ Foo$ $\langle proof \rangle$
next
  case $(Bar\ x)$
  note $\langle P\ x \rangle$
  then show $P\ (Bar\ x)$ $\langle proof \rangle$
qed

Basic observations

- Checking specifications can take considerable time,
  but the result is determined syntactically.
- Checking proofs takes 95% of the time,
  but proofs are irrelevant (in Isabelle/Pure).
- Checking terminal justifications takes 95% of proof time,
  but Isar structure does not really care.

Principles of "asynchronous" proof processing

- Commands (transactions) are explicitly identified (unique labels)
- States (after successful transactions) may be addressed explicitly
- Structural dependencies are observed, e.g.
  - sequential composition of consecutive transactions (default)
  - parallel composition of independent branches
  - nesting due to logical block structure
- Fine-grained result state of transactions, e.g.
  - *unprocessed*
  - *syntax-checked*
  - *proof-checked*
- Dynamic message model, e.g. progress reports

Example: irrelevant proofs

lemma $[simp]$: $attributes\ (Val\ (att,\ text)) = att$

by $(simp\ add\colon attributes\text{-}def)$

lemma $[simp]$: $attributes\ (Env\ att\ dir) = att$

by $(simp\ add\colon attributes\text{-}def)$

lemma $[simp]$: $attributes\ (map\text{-}attributes\ f\ file) = f\ (attributes\ file)$

by $(cases\ file)\ (simp\text{-}all\ add\colon attributes\text{-}def\ map\text{-}attributes\text{-}def\ split\text{-}tupled\text{-}all)$

lemma $[simp]$: $map\text{-}attributes\ f\ (Val\ (att,\ text)) = Val\ (f\ att,\ text)$

by $(simp\ add\colon map\text{-}attributes\text{-}def)$

lemma $[simp]$: $map\text{-}attributes\ f\ (Env\ att\ dir) = Env\ (f\ att)\ dir$

by $(simp\ add\colon map\text{-}attributes\text{-}def)$

Example: derived specifications

inductive
$transition :: file \Rightarrow operation \Rightarrow file \Rightarrow bool$
$(\text{-} \longrightarrow\!\!\!\!\!\!\cdot\ \text{-}\ [90, 1000, 90]\ 100)$
where
$read$:
$access\ root\ path\ uid\ \{Readable\} = Some\ (Val\ (att, text)) \Longrightarrow$
$root\ -(Read\ uid\ text\ path)\rightarrow\ root\ |$
$write$:
$access\ root\ path\ uid\ \{Writable\} = Some\ (Val\ (att, text'')) \Longrightarrow$
$root\ -(Write\ uid\ text\ path)\rightarrow\ update\ path\ (Some\ (Val\ (att, text)))\ root\ |$
$chmod$:
$access\ root\ path\ uid\ \{\} = Some\ file \Longrightarrow$
$uid = 0 \vee uid = owner\ (attributes\ file) \Longrightarrow$
$root\ -(Chmod\ uid\ perms\ path)\rightarrow\ update\ path$
$(Some\ (map\text{-}attributes\ (others\text{-}update\ (K\text{-}record\ perms))\ file))\ root\ |$
...

$\langle monotonicity\ proof \rangle$

$\langle main\ proof \rangle$

Example: sub-structured proofs

theorem *transition-uniq*:
assumes $root'$: $root\ -x\longrightarrow root'$ **and** $root''$: $root\ -x\longrightarrow root''$
shows $root' = root''$ **using** $root''$

proof *cases*
case *read*
with $root'$ **show** *?thesis* **by** *cases auto*
next
case *write*
with $root'$ **show** *?thesis* **by** *cases auto*
next
case *chmod*
with $root'$ **show** *?thesis* **by** *cases auto*
next
...
qed

Main agents: provers, editors, users

Provers

- Attempt to cover broad range of existing provers:
  Isabelle, Mizar, Coq, Matita, etc.
- Define general principles,
  but do not set particular features in stone
- Implementation options:
  1. full version: native support of asynchronous checking
     (including parallel processing etc.)
  2. restricted version: fit unchanged systems into the model
  3. mixed version: additional support by "middle ware"

Editors

- Open-mindedness to cover broad range of editing environments: web interfaces, Emacs, jEdit, etc.
- Down-scaled demands on specific features:
  - No locking of text regions (only highlighting)
  - Undo/redo follows editor view, not prover
  - Even less "structure editing"
  - Replace responses (warnings, errors) by in-text annotation
  - Abolish proof state buffer!?
- Convergence of exiting efforts on building post-Proof-General interfaces?

Users

- User types: address beginners and experts alike
- User empowerment: more freedom in out-of-order editing, top-down development, composing outlines, multiple views
- User groups: support collaborative editing (How?)

Further issues:

- Generalizing multi-threaded / multi-viewed document processing towards multi-user processing.
- Integration with (centralized or distributed) repositories.